Multistage Stochastic Mixed Integer Programming using DSP

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Multistage Stochastic Mixed Integer Programming

Let $\alpha_k(n)$ denote the k^{th} ancestor of node n, $t(n)$ the time period of node n in the scenario tree, and p_n the probability associated with node n.

$$z := \min_{\mathbf{y}_{gn}} \sum_{n=1}^{N} p_n \sum_{g=1}^{G} \mathbf{d}_{gn}^{\top} \mathbf{y}_{gn} \quad (1a)$$

$$\text{s.t. } \mathbf{W}_{g0}\mathbf{y}_{g0} = \mathbf{h}_{g0} \quad \forall g \in \{1,\dots,G\}, \quad (1b)$$

$$\mathbf{T}_{g,\alpha_k(n)}\mathbf{y}_{g,\alpha_k(n)} + \mathbf{W}_{gn}\mathbf{y}_{gn} = \mathbf{h}_{gn} \quad \forall g \in \{1,\dots,G\}, \quad n \in \{1,\dots,N\}, k \in \{1,\dots,t(n)\}, \quad (1c)$$

$$\mathbf{y}_{gn} \in \mathbf{Y} \quad \forall g \in \{1,\dots,G\}, n \in \{1,\dots,N\}. \quad (1d)$$

Challenges: Complexity: NP-hard (generally); Curse of dimensionality; etc.

Applications

- Unit Commitment & Economic Dispatch:

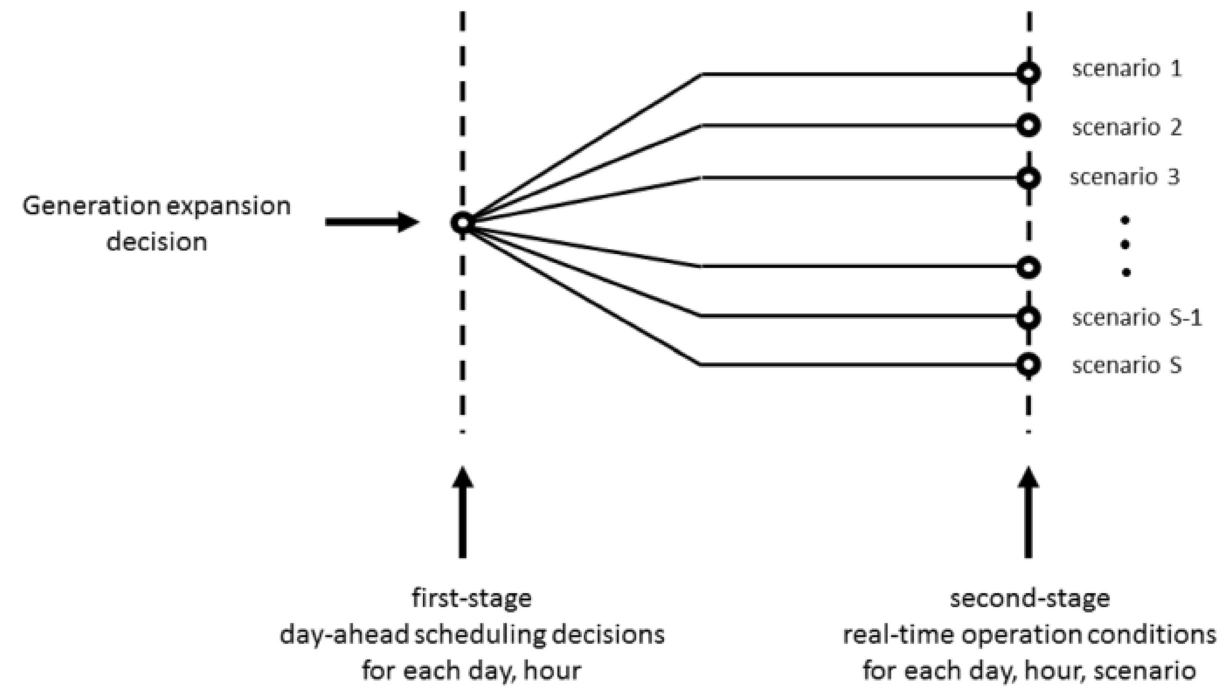


Figure 1:A. Schwele, et. al. (2020). *Do UC constr.'s affect gen. exp. planning? Scal. stoch. model*

- Other applications: portfolio management, resource allocation, disaster mitigation.

Motivation: When & How?

When does a multistage extension make sense?

- Sequential decision making; handling uncertainty dynamically; minimizing costs over multiple time periods.

Aim: implement a parallel algorithm to solve multistage SMIPs.

- Using Julia and a parallel decomposition algorithm speeds up computations.
- Using a multistage model rather than a two-stage model improves solution quality and allows for uncertainty to be handled dynamically.
- Motivated by increase in computing power & Julia modeling languages.

Decomposition for Scenario Trees/Lattices

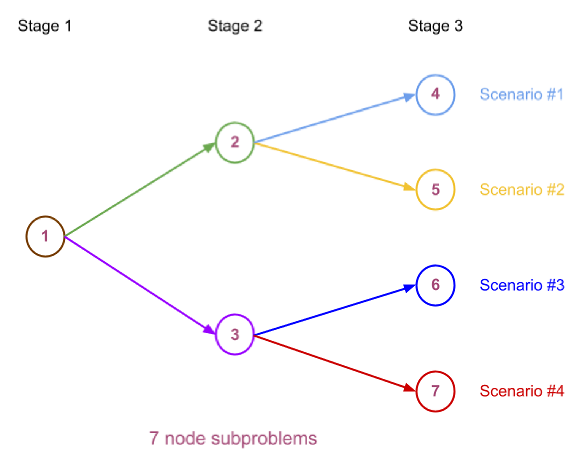


Figure 2:3-stage Scenario Tree

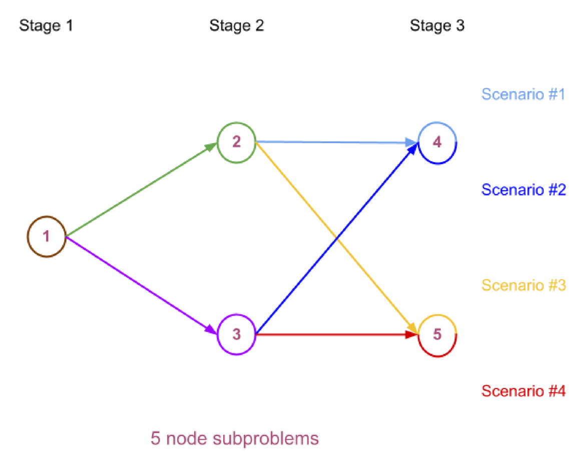


Figure 3:3-stage Scenario Lattice

- Discrete-time stochastic process.
- Markovian (memoryless) process.

We can decompose the problem by *scenarios* or by *nodes*.

Decomposition of Structured Programs (DSP)

- DSP is a parallel decomposition MIP solver.
- Can implement decomposition methods for structured MIPs.
- **Multistage SMIP has a block-diagonal structure that makes it easily parallelizable.**

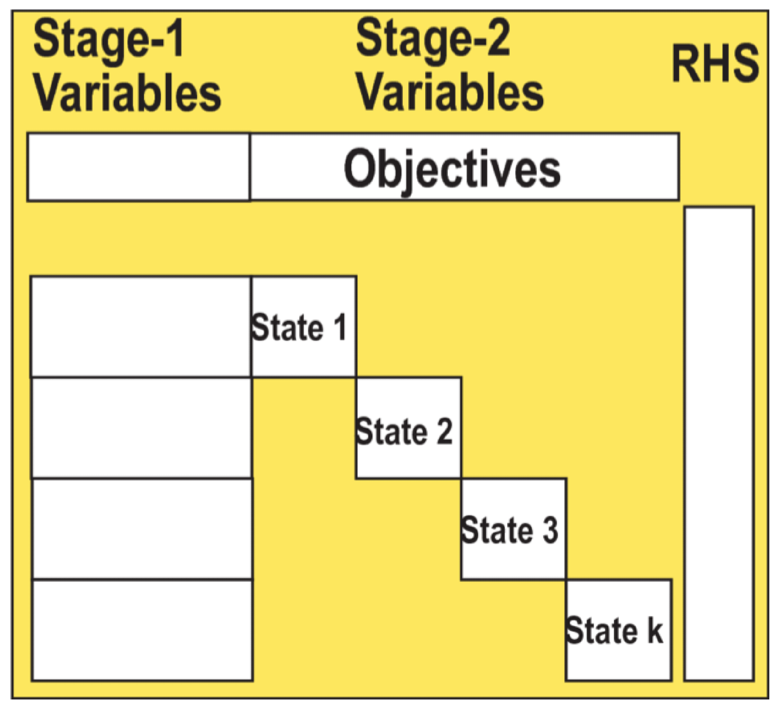
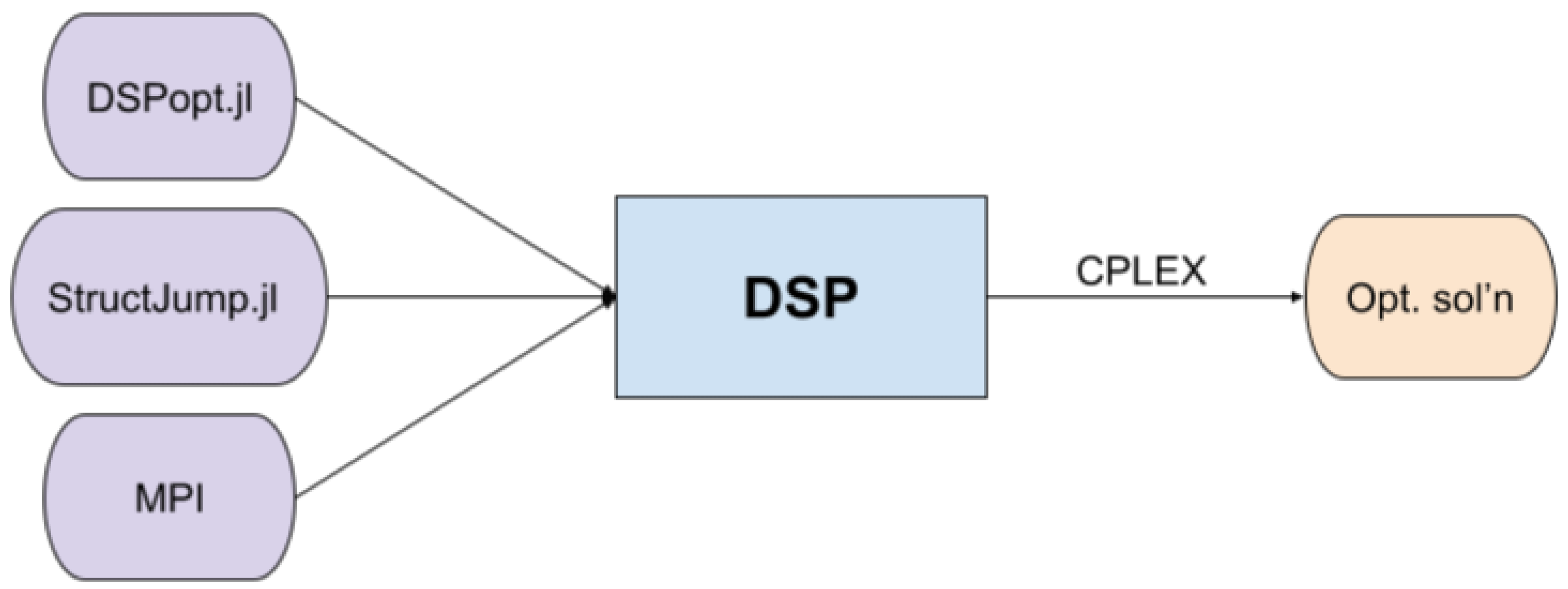
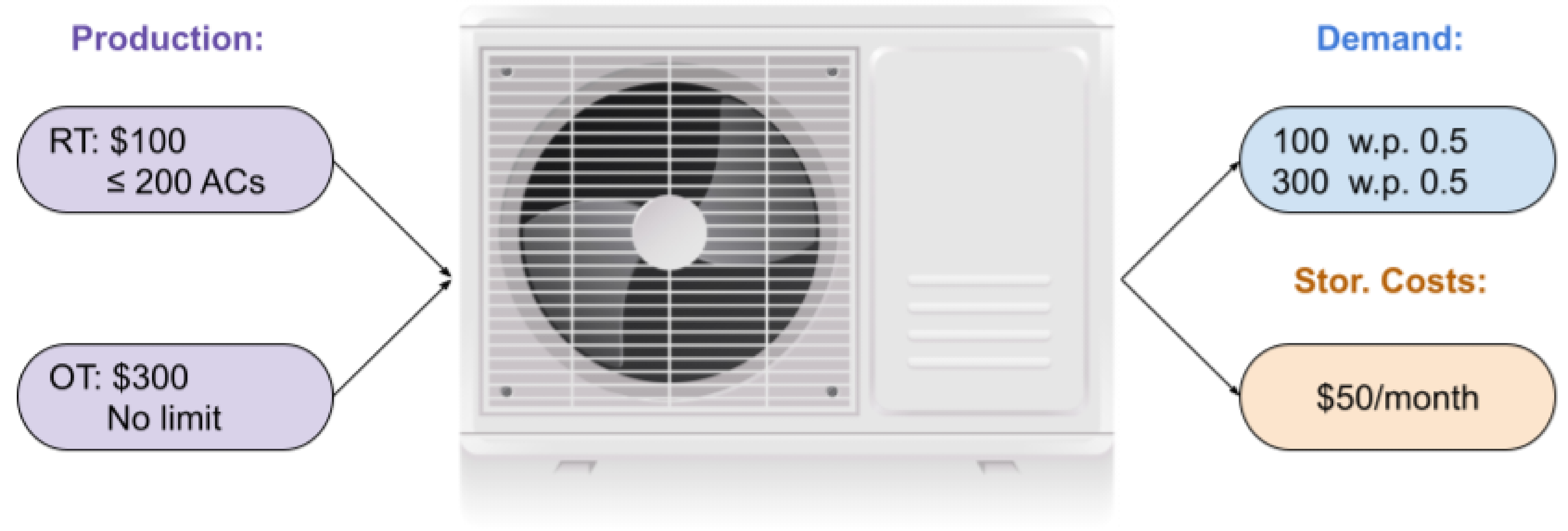


Figure 4:R.M. Freund (2004). *Benders' Decomp. Methods for Struct. Opt., incl. Stoch. Opt.*



Goal: Encode optimization problem so that it can be read by *StructJuMP* and fed into *DSPopt* to solve in parallel.

Example: T-stage AC production problem



How many ACs do we produce, with which type of labor, in each time period, to meet demand while minimizing production cost?

Code is available here: `https://github.com/kibaekkim/DSPopt.jl/tree/ra/multistage/examples/multistage`.

Computational Results

The following computations were run on an Argonne compute node with CPU: 1X AMD EPYC 7453 28-Core CPU, 256GB Memory, running Ubuntu 20.04 as its OS.

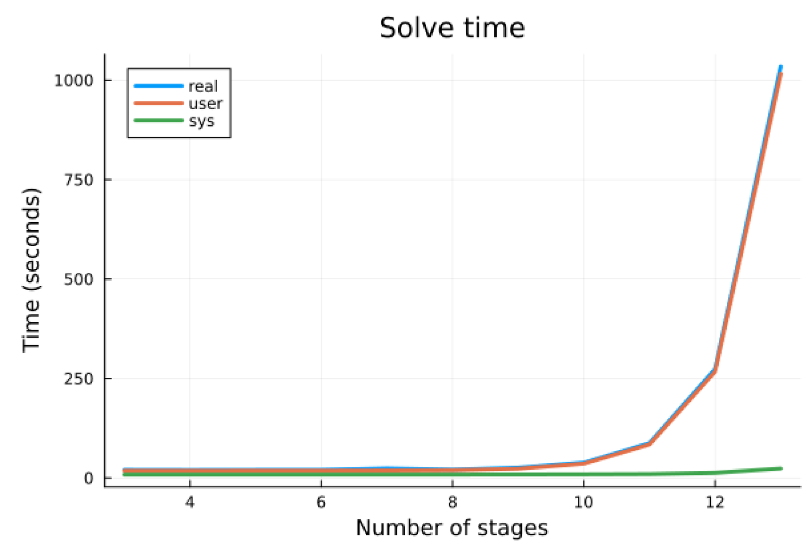


Figure 5:Solve time for Scenario Tree

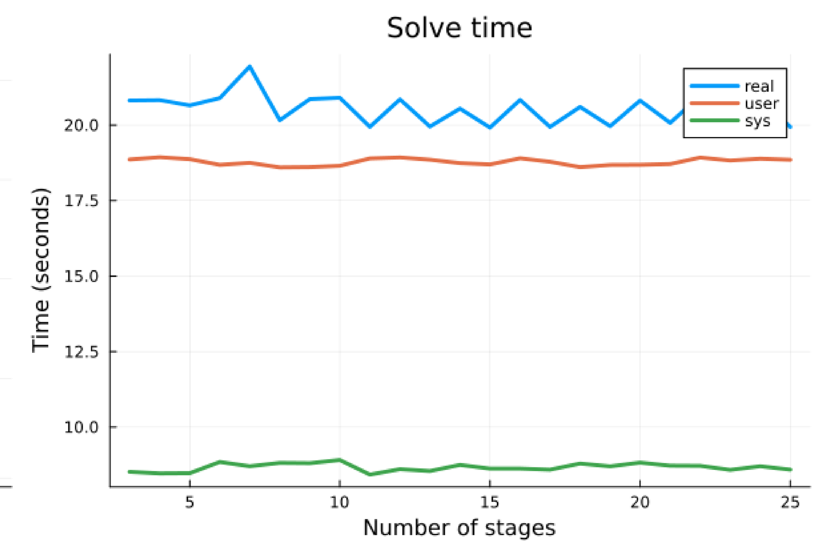


Figure 6:Solve time for Scenario Lattice

Future directions:

- Generate cuts to decrease the feasible region & speed up computations.
- Apply breakstages (Zou, J. et al. (2018)).

References

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- Kim, K. & Zavala, V.M. (2018). *Algorithmic innovations and software for the dual decomposition method applied to stochastic mixed-integer programs*. Math. Prog. Comp. 10, 225-266.
- Zou, J., Ahmed, S., & Sun, X. A. (2018). *Multistage Stochastic Unit Commitment using SDDiP*. IEEE Transactions on Power Systems, 34(3), 1814-1823.

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