OPTIMIZATION OF CONTROLLED FREE-TIME SWEEPING PROCESSES

Giovanni Colombo, Boris S. Mordukhovich, Dao Nguyen and Trang Nguyen

WAYNE STATE UNIVERSITY

Introduction

Sweeping process models were introduced by Jean-Jacques Moreau in the 1970s to describe dynamical processes arising in elastoplasticity and related mechanical areas. Many important results have been obtained on necessary optimality conditions for controlled sweeping processes with valuable applications to robotics, traffic equilibria, economics, and other fields of engineering and applied sciences. This project is devoted to establishing necessary optimality conditions for free-time sweeping processes with a non-smooth perturbation with respect to the control variable and shows its practical applications in biotechnology and medicine at the nanoscale.

Problem and Formulations

Consider the problem (P)

$$\text{minimize} \quad J[x,u,T] := \varphi(x(T),T),$$

over the control function $u(\cdot)$ and the corresponding trajectories $x(\cdot)$ satisfying

$$\begin{cases} \dot{x}(t) \in -N\big(x(t);C(t)\big) + g\big(x(t),u(t)\big) \text{ a.e. } t\in[0,T],\\ x(0)=x_0\in C\subset\mathbb{R}^n,\\ u(t)\in U\subset\mathbb{R}^d \text{ a.e. } t\in[0,T],\\ (x(T),T)\in\Omega_x\times\Omega_T\subset\mathbb{R}^n\times[0,\infty),\end{cases}$$

where the sweeping set $C(t)$ is given by

$$C(t) := \bigcap_{j=1}^{s} C^j(t)$$

with

$$\begin{cases} C^j(t) := \big\{x\in\mathbb{R}^n \big|\ \langle x_*^j(t),x\rangle \le c_j(t)\big\},\\ \|x_*^j\| = 1,\ j=1,\dots,s.\end{cases}$$

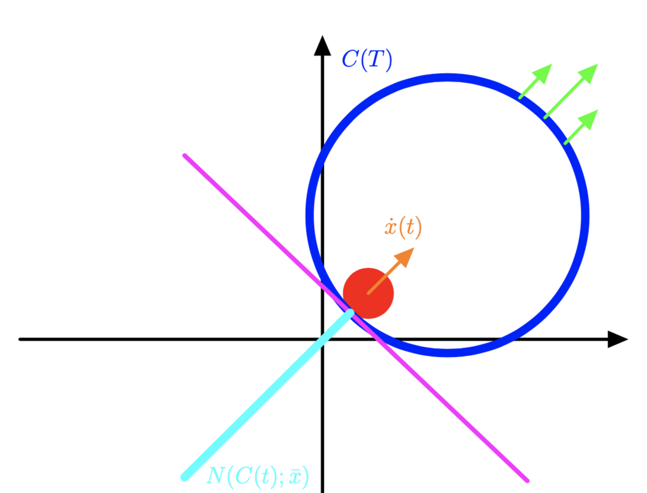

Figure: Sweeping process model

Assumption

Consider the index set

$$I(t,x) := \{j\in\{1,\dots,s\} \,|\, \langle x_*^j(t),x\rangle = c_j(t)\}.$$

There are some *standing assumptions* as follows:

(H1) The set $U\neq\emptyset$ is a compact set in $\mathbb{R}^d$, $x_*^j(\cdot)$, and $c_j(\cdot)$ are Lipschitz with a commom Lipschitz constant L.

(H2) There exists a continuous function $\vartheta : [0,T]\to\mathbb{R}$ such that $\sup_{t\in[0,T]}\vartheta(t)<0$ and

$$\{x\in\mathbb{R}^n | \langle x_*^j(t),x\rangle - c_j(t) < \vartheta(t), i=1,\dots,s\}\neq\emptyset$$

for all $t\in[0,T]$.

(H3) $g\colon\mathbb{R}^n\times U\to\mathbb{R}^n$ is Lipschitz continuous with respect to x uniformly on U whenever x belongs to a bounded subset of $\mathbb{R}^n$ and satisfie

$$\|g(x,u)\| \le \beta\big(1+\|x\|\big) \quad \text{for all} \quad u\in U,$$

with some positive constant β.

(H4) The set $\Omega_x\times\Omega_T$ is closed around $(\bar{x}(\overline{T}),\overline{T})$.

Methods

We use the method of discrete approximations with establishing its well-posedness and strong convergence, enabling them to handle the non-smoothness of the perturbation with respect to the control function. Firstly, we develop the collections of necessary optimality conditions for local minimizes of ***discrete approximation problem*** (P_k):

$$\text{minimize} \quad J_k[x^k,u^k,T_k] := \varphi(x_k^k,T_k) + (T_k-\overline{T})^2 + \sum_{i=0}^{k-1}\int_{t_i^k}^{t_{i+1}^k}\left(\left\|\frac{x_{i+1}^k - x_i^k}{h^k} - \dot{\bar{x}}(t)\right\|^2 + \left\|u_i^k - \bar{u}(t)\right\|^2\right)dt.$$

using advanced tools of first-order and second-order variational analysis and generalized differentiation. Then, we derive the necessary optimality conditions for local minimizers of the original sweeping optimal control problem by passing to the limit as $k\to\infty$ in the optimality conditions for problems (P_k).

Necessary Optimality Conditions for the Controlled Sweeping Process

Let $(\bar{x}(t),\bar{u}(t),\overline{T})$, $0\le t\le\overline{T}$ be a relaxed $W^{1,2}\times L^2\times\mathbb{R}_+$-local minimizer to the problem (P). Then there exist a multiplier $\mu\ge0$, $\gamma_> = (\gamma_>^1,\dots,\gamma_>^s)\in C^*([0,\overline{T}];\mathbb{R}^s)$ and $\gamma_0=(\gamma_0^1,\dots,\gamma_0^s)\in C^*([0,\overline{T}];\mathbb{R}^s)$, $p(\cdot)\in W^{1,2}([0,\overline{T}];\mathbb{R}^n)$ and $q(\cdot)\in BV([0,\overline{T}];\mathbb{R}^n)$ such that

1. $-\dot{\bar{x}}(t) = \sum_{j=1}^{s}\eta^j(t)x_*^j(t) - g\big(\bar{x}(t),\bar{u}(t)\big)$ for a.e. $t\in[0,\overline{T})$,
2. $(-\dot{p}(t),\psi(t))\in\operatorname{co}\partial\langle q(t),g\rangle(\bar{x}(t),\bar{u}(t))$, with $\psi(t)\in\operatorname{co}N(\bar{u}(t);U)$ for a.e. $t\in[0,\overline{T}]$, where
$$q(t) = p(t) - \int_{(t,\overline{T}]}\sum_{j=1}^{s} d\gamma^j(\tau)x_*^j(\tau),$$
3. $\langle\psi(t),\bar{u}(t)\rangle = \max_{u\in U}\langle\psi(t),u\rangle$ for a.e. $t\in[0,\overline{T}]$,
4. $\langle x_*^j(t),\bar{x}(t)\rangle < c_j(t) \Longrightarrow \eta^j(t)=0$ and $\eta^j(t)>0\Longrightarrow \langle x_*^j(t),q(t)\rangle = 0$ for a.e. $t\in[0,\overline{T}]$ and all $j=1,\dots,s$ provided that LICQ at $\bar{x}(t)$,
5. $(-p(\overline{T}) - \sum_{j\in I(\overline{T},\bar{x}(\overline{T}))}\eta^j(\overline{T})x_*^j(\overline{T}),\bar{H})\in\mu\partial\varphi(\bar{x}(\overline{T}),\overline{T})$, and $\eta^j(\overline{T})>0\Longrightarrow j\in I(\overline{T},\bar{x}(\overline{T}))$, where $\bar{H} := \frac{1}{\overline{T}}\int_0^{\overline{T}}\langle p(t),\dot{\bar{x}}(t)\rangle dt$,
6. $\langle x_*^j(\overline{T}),\bar{x}(\overline{T})\rangle < c_j(\overline{T}) \Longrightarrow \eta^j(\overline{T})=0$,
7. If $t\in[0,\overline{T})$ and $\langle x_*^j(t),\bar{x}(t)\rangle < c_j$, for all $j=1,\dots,s$, then there exists a neighborhood V_t of t in $[0,\overline{T})$ such that $\gamma_0^j(V)=\gamma_>^j(V)=0$ for all the Borel subsets V of V_t,
8. $(\mu,p,\|\gamma_0\|_{TV},\|\gamma_>\|_{TV})\neq0$, with $\operatorname{supp}(\gamma_>)\cap\operatorname{int}(E_0)=\emptyset$.

Email: ha2288@wayne.edu, web: https://clasprofiles.wayne.edu/profile/ha2288

Applications

In order to enhance the effectiveness of numerous drugs and facilitate the utilization of novel drugs and therapies, we applied the necessary optimality conditions to the model of the motion of nanoparticles in straight tubes, which is treated as a sweeping optimal control problem

$$\text{minimize} \quad J[x,u,T] := \frac{1}{2}\|x(T)\|^2 + \frac{1}{2}T^2,$$

taking into account all the interactions between nanoparticles and with walls and obstacles.

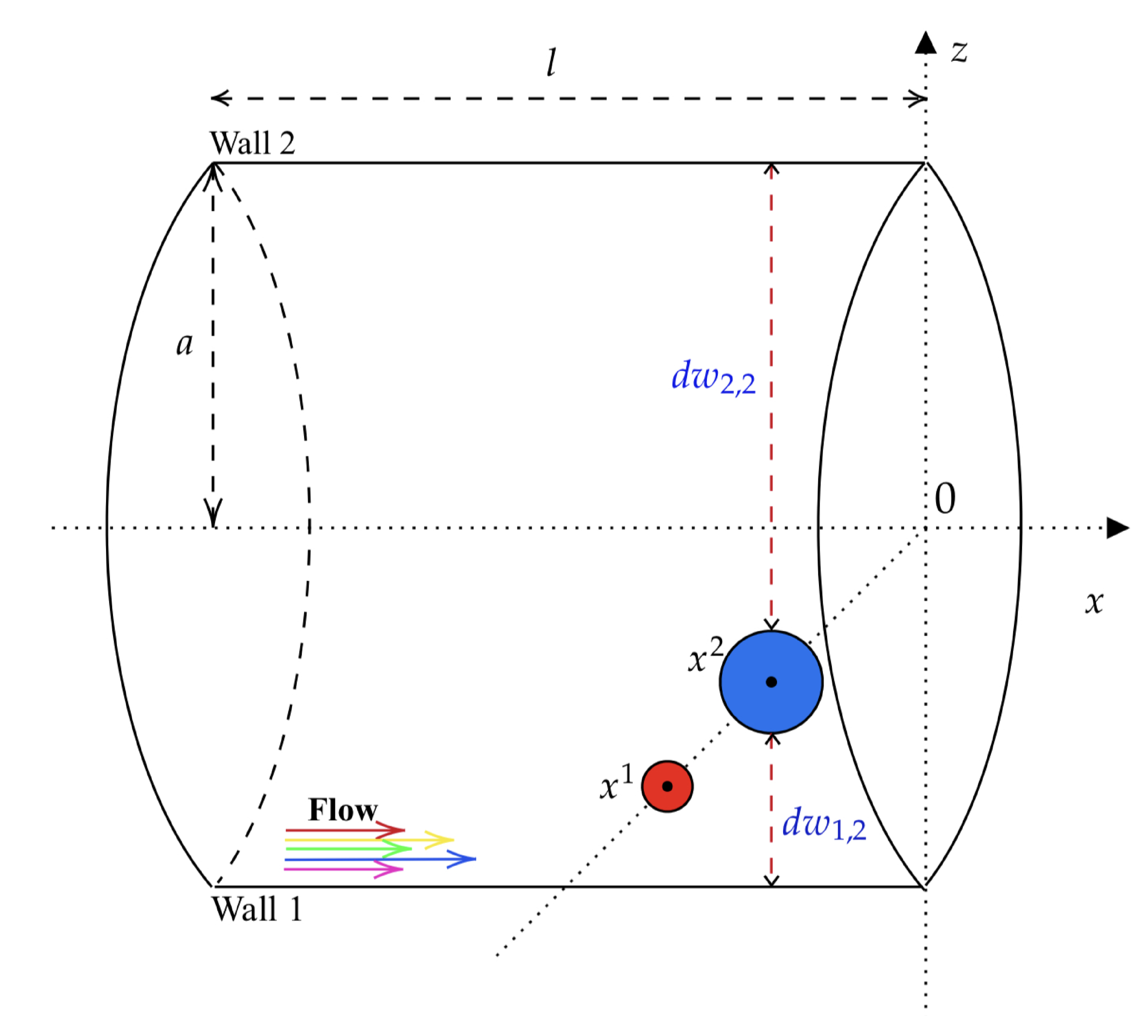

Figure: Nanoparticles in a straight tube

References

[1] T. H. Cao, N. T. Khalil, B. S. Mordukhovich, D. Nguyen, and T. Nguyen.
Optimization of controlled free-time sweeping processes with applications to marine surface vehicle modeling.
IEEE Syst. Cont. Lett., 6:782 – 787, June 2021.

[2] G. Colombo, B. S. Mordukhovich, D. Nguyen, and T. Nguyen.
Optimization of controlled free-time sweeping processes.
to appear.

Curent research

Optimization of perturbed sweeping processes with constrained controls and a time-delayed system.