Dynamics of a Multi-Fluid System

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Abstract

- A long-wave asymptotic model is used to study a three-layer falling film consisting of three immiscible fluids with identical densities but different viscosities inside a vertical tube.
- The impact of viscosity and thickness on the linear growth of the interfacial disturbances is investigated, when disturbances are small.
- The growth rate of the disturbances is computed by solving an eigenvalue problem, which helps determine where the system is unstable to small amplitude disturbances.
- Nonlinear evolution equations are derived to investigate the dynamics when the disturbances are not small.

Introduction

- Viscous film flows in tubes have been at the center of many experiments and studies as they are present in many applications such as biological processes (human airways) and industrial processes (pipes).

Previous Work

- Rayleigh (1878) studied the instability of an infinite liquid jet for small disturbances and its mode of disintegration.

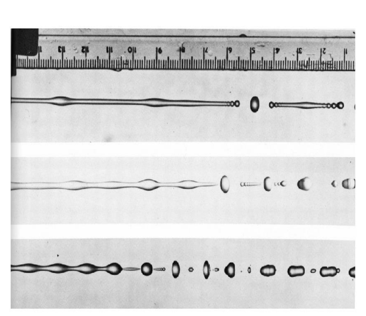


Figure 4.1 The break-up of a jet of water, as different wavelengths are excited. (From Van Dyke, 1982, Fig. 122.)

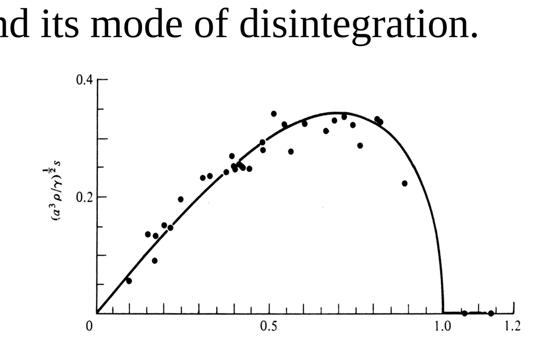


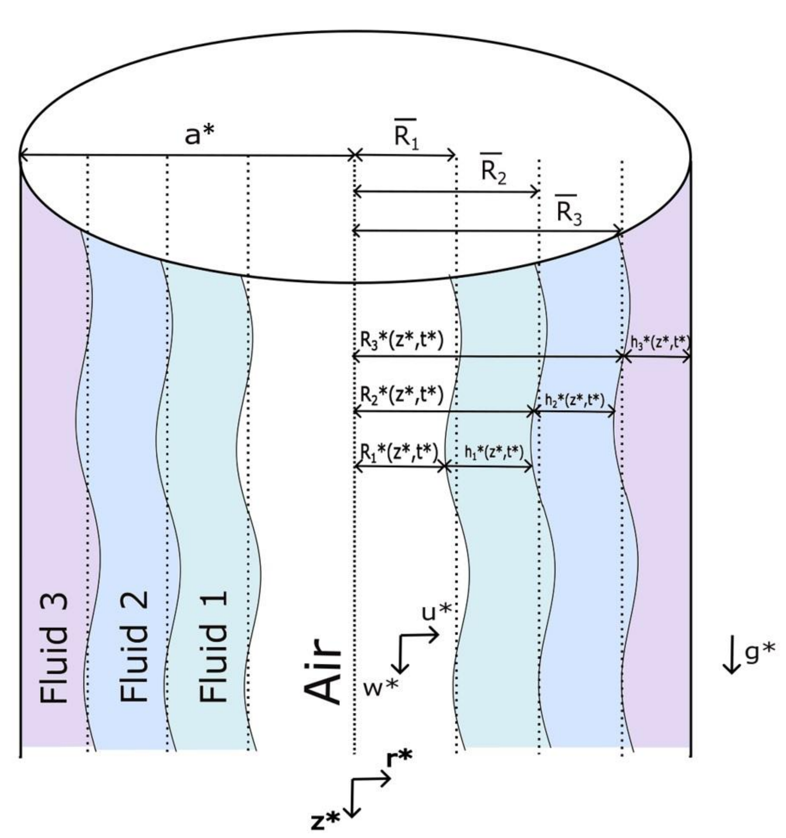
Figure 4.2 Graph of the dimensionless growth rate $(a^3\rho/\gamma)^{1/2}s$ against the dimensionless wavenumber α for axisymmetric capillary modes ($n = 0$). The curve denotes Rayleigh's theoretical results and the points the experimental results measured by Donnelly & Glaberson (1966).

Introduction to Hydrodynamic Stability by P.G. Drazin

- Tomotika (1935) investigated the stability of a long cylindrical thread of a viscous liquid surrounded by another viscous fluid in the limit of small disturbances. He found a relation between the maximum growth rate and its dependence on viscosity ratio.
- Goren (1961) considered the linear stability of viscous liquid thread coating the inside/outside of a cylindrical tube.
- Hammond (1983) studied the nonlinear analysis for the adjustment of initially uniform film of viscous fluid within a cylindrical pipe, which led to the formation of collars and lobes.
- Ogrosky (2021) used a long-wave model to study a two-layer falling film consisting of two immiscible viscous fluids with identical densities but different viscosities lining the interior of a vertical tube. Both linear and nonlinear stability analysis were conducted.

Methods

- The impact of viscosity, surface tension and base flow due to gravity on the linear stability of the momemtum equations (Navier Stokes equations), appropriate when the disturbances of the interfaces between the liquid layers are small, is investigated.
- A long-wave asymptotic model is used to study the dynamics of a three-layer flow with two interfaces between the layers and one free surface between the innermost fluid and the passive air core.
- The growth rate of the disturbances is computed by solving an eigenvalue problem, which enables me to determine where in parameter space the flow system is unstable to small disturbances.
- Nonlinear evolution equations have also been derived and are used to investigate the dynamics of the multi-layer fluid system when the disturbances are not small.
- The nonlinear equations are solved numerically using the method of lines. The purpose here is to determine whether small disturbances grow without bound or they saturate as finite amplitude waves.

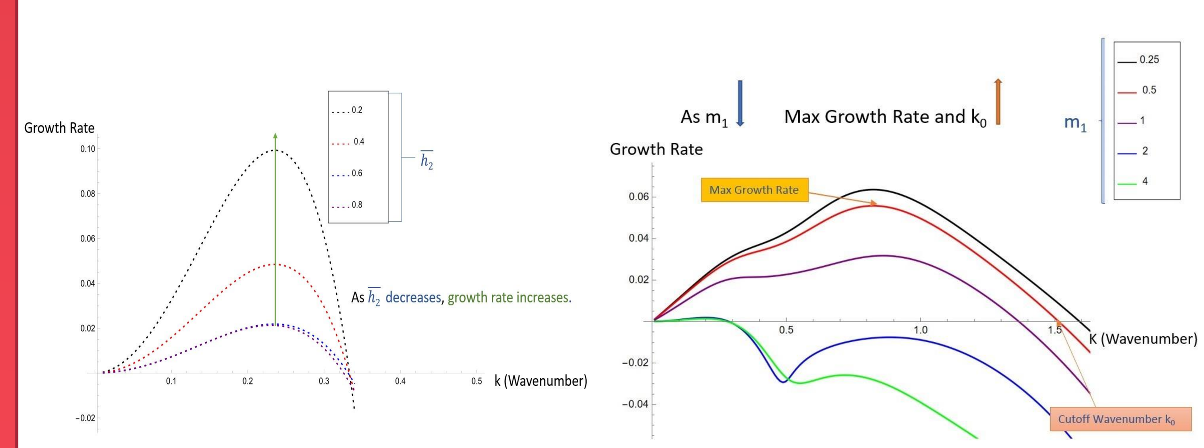


Three Fluids In a Cylinder With Passive Core

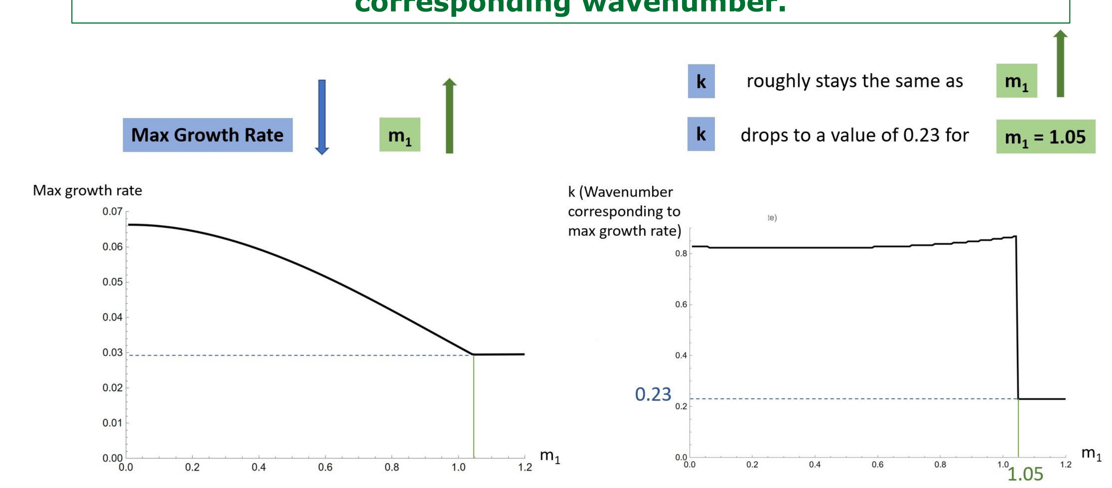
Results

Linear Analysis

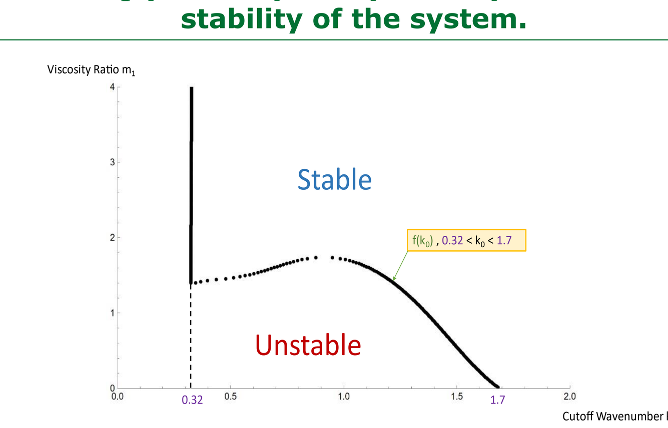
Influence of $\overline{h_2}$ (thickness of Fluid 2) on the growth rate.	Influence of $m_1 = \mu_3^*/\mu_1^*$ (viscosity ratio) on the growth rate & cutoff wavenumber.



Effects of m_1 (viscosity ratio) on the max growth rate and its corresponding wavenumber.

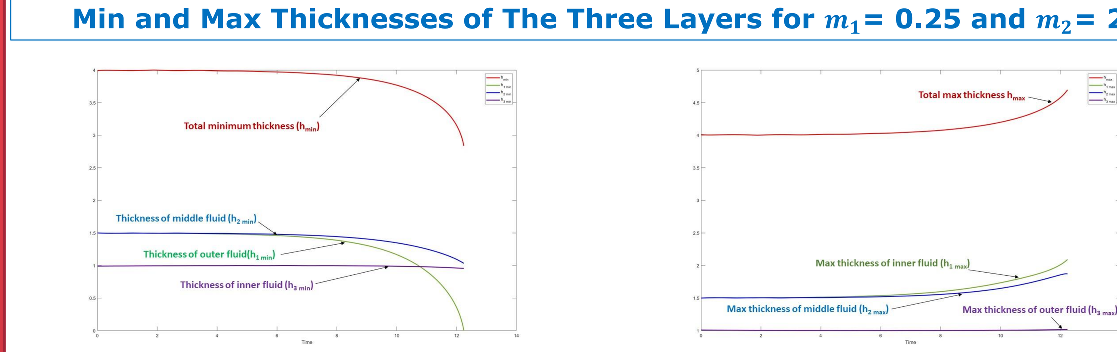


Influence of m_1 (viscosity ratio) and k (wavenumber) on the stability of the system.

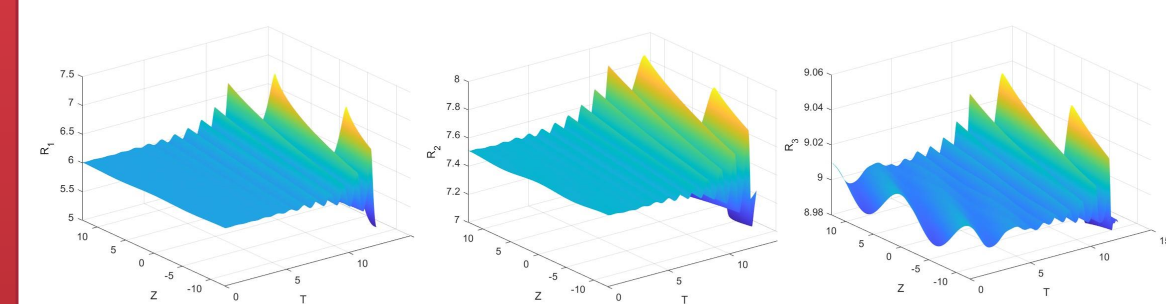


Nonlinear Analysis

Min and Max Thicknesses of The Three Layers for $m_1 = 0.25$ and $m_2 = 2$.



Surface Plots of The Free Surface (R_1) and Interfaces (R_2 and R_3)



Conclusions/Significance

Linear Analysis

- Influence of $\overline{h_2}$, the thickness of fluid 2, over on the growth rate: As $\overline{h_2}$ decreases the growth rate increases.
- Influence of viscosity ratio m_1 on growth rate: As m_1 decreases, the max growth rate and cutoff wavenumber of the disturbances increase.
- There exists a marginal wavenumber for which the system is stable for any wavenumber greater than that number.

Nonlinear Analysis

- For certain parameter values, preliminary numerical simulations suggest plug formation.

Future Directions

- Examine influence of other parameters, such as surface tension, on the stability characteristics.
- Use the method of lines for the nonlinear analysis.
- Use the same approach for a system consisting of two concentric cylinders i.e., use a long wave model to study the linear stability and solve the nonlinear evolution equations numerically.
- Conduct theoretical studies for arbitrary wavelengths.

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